

# Modern Developments in Geometry and Higher Structures

## June 9-13, 2025

## <u>About the workshop</u>

Recent breakthroughs in higher category theory and homotopy theory have allowed mathematicians to capture subtle geometric phenomena, which has led to new connections between derived algebraic geometry, algebraic K-theory, and arithmetic. This workshop will bring together experts working on the applications of modern techniques from higher algebra to arithmetic, algebraic, and analytic geometry, in particular algebraic cycles, constructible sheaves, and the six functors formalism.

## <u>Organizers</u>

Ettore Aldrovandi, Florida State University Piotr Pstragowski, Kyoto University Niranjan Ramachandran, University of Maryland

## <u>Participants</u>

Toni Annala, University of Chicago Benjamin Antieau, Northwestern University Sanath Devalapurkar, Harvard University Carlos di Fiore, Universidad de Buenos Aires Dennis Eriksson, Chalmers University Martin Gallauer, University of Warwick David Gepner, Johns Hopkins University Rok Gregoric, Johns Hopkins University Peter Haine, University of California, Berkeley Alice Hedenlund, NTNU Lars Hesselholt, Copenhagen & Nagoya Universities Ryomei Iwasa, CNRS, Paris Saclay

Dmitry Kaledin, HSE University Adeel Khan, Academia Sinica Lucas Mann, Universität Münster Akhil Mathew, University of Chicago Fernando Muro, University of Seville Kate Ponto, University of Kentucky Arpon Raksit, MIT Maxime Ramzi, Universität Münster Germán Stefanich, MPIM Goncalo Tabuada, University of Warwick Lucy Yang, Columbia University Bogdan Zavyalov, IAS/Princeton University



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DEPARTMENT OF MATHEMATICS

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## Schedule at a Glance

8.00	Monday	Tuesday	Wednesday	Thursday	Friday
0:00	Ducaltfact			_	
0.00	Breaklast		Breakfast		
9.00	Antieau	Breakfast	Zavyalov	Breakfast	Breakfast
10.00		Gallauer		Annala	Raksit
10.00	Coffee Break		Kaledin		
11:00	Mann	Coffee Break		Coffee Break	Coffee Break
		Gepner	Coffee Break	Muro	Mathew
12:00	Lunch		Tabuada		
12.00		Lunch	-	Lunch	
13:00		Lunch		-	
	Devalapurkar				
14:00					
	Yang	Haine		Stefanich	
15:00	10005				
	Coffee Break	di Fiore		Hedenlund	
16:00	Khan			-	
		Coffee Break		Coffee Break	
17:00		Hesselholt		Gregoric	
18:00					

## Workshop Overview

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## Organizing committee

ETTORE ALDROVANDI, Florida State University PIOTR PSTRAGOWSKI, Kyoto University NIRANJAN RAMACHANDRAN, University of Maryland

## Workshop Schedule

### Monday, June 9, 2025

- 8:45 9:00 DORON LEVY (University of Maryland/Director, Brin MRC) Opening
- 9:00 10:00 BENJAMIN ANTIEAU (Northwestern University) E-infinity Descendability
- 10:00 10:30 Coffee Break
- 10:30 11:30 LUCAS MANN (Universitat Munster) 6-Functor Formalisms and Duality
- 11:30 1:00 Lunch
- 1:00 2:00 SANATH DEVALAPURKAR (Harvard University) Spherochromatism in Geometric Representation Theory
- 2:15 3:15 LUCY YANG (Columbia University) Involutions and the Brauer Group in Derived Algebraic Geometry
- 3:15 3:45 Coffee Break
- 3:45 4:45 ADEEL KHAN (Academia Sinica) Microlocal Categorical Sheaves on Shifted Symplectic Spaces
- 6:30 8:30 Conference Dinner

### TUESDAY, JUNE 10, 2025

- 9:30 10:30 MARTIN GALLAUER (University of Warwick) Around Local Monodromy
- 10:30 11:00 Coffee Break
- 11:00 12:00 DAVID GEPNER (Johns Hopkins University) Lax Higher Category Theory
- 12:00 12:15 GROUP РНОТО
- 12:15 2:00 Цинсн
- 2:00 3:00 PETER HAINE (University of California, Berkeley) The Proetale Homotopy Type
- 3:15 4:15 CARLOS DI FIORE (Universidad de Buenos Aires) An Exceptional Inverse Image Functor for Sheaves of Categories
- 4:15 4:45 Coffee Break
- 4:45 5:45 LARS HESSELHOLT (University of Copenhagen/Nagoya University) Condensed Anima

## WEDNESDAY, JUNE 11, 2025

#### 8:30 - 9:00 Breakfast

- 9:00 10:00 BOGDAN ZAVYALOV (IAS/Princeton University) Poincare Duality in Abstract 6-Functor Formalisms
- 10:00 11:00 DMITRY KALEDIN (Higher School of Economics) How to Enhance Categories, and Why
- 11:00 11:30 Coffee Break
- 11:30 12:30 GONCALO TABUADA (University of Warwick) Grothendieck Classes of Twisted Symplectic Grassmannians

## Thursday, June 12, 2025

9:00 - 9:30	Breakfast
9:30 - 10:30	TONI ANNALA (University of Chicago) Motivic Power Operations at the Characteristic
10:30 - 11:00	Coffee Break
11:00 - 12:00	FERNANDO MURO (University of Seville) Geometry of Singularities and Higher Structures
12:00 - 2:00	Lunch
2:00 - 3:00	GERMAN STEFANICH (Max-Planck Institute for Mathematics) Categorification of Six Functor Formalisms
3:15 - 4:15	ALICE HEDENLUND (Norwegian University of Science and Technology) The Synthetic Hilbert Additive Group Scheme
4:15 - 4:45	Coffee Break
4:45 - 5:45	ROK GREGORIC (Johns Hopkins University)

## FRIDAY, JUNE 13, 2025

- 9:00 9:30 Breakfast
- 9:30 10:30 ARPON RAKSIT (Massachusetts Institute of Technology)
- 10:30 11:00 Coffee Break
- 11:00 12:00 AKHIL MATHEW (University of Chicago) Sheared Prismatization

## Abstracts of talks

## E-infinity Descendability

## BENJAMIN ANTIEAU

Northwestern University

Monday, June 9, 2025 @ 9:00 AM

I will report on joint work with German Stefanich which introduces the notion of E-infinity descendable morphisms of E-infinity rings. A priori, this is a strengthening of Akhil Mathew's notion of descendable morphisms. I will give several interesting classes of descendable morphisms which are in fact E-infinity descendable and I will pose several open problems.

## 6-Functor Formalisms and Duality

#### LUCAS MANN

#### Universitat Munster

#### Monday, June 9, 2025 @ 10:30 AM

Grothendieck's six operations have proven to be an invaluable tool in many geometric contexts and allow one to abstract results like Kenneth formula and Poincare duality, thus providing a universal and clean framework for them. In recent joint work with Heyer we lay the foundations for a systematic study of abstract 6-functor formalisms through the associated 2-category of kernels, a construction that first appeared in Fargues–Scholze's work and is based on previous work by Lu– Zheng. Even though this 2-category is easy to define, it provides a powerful new perspective on the 6-functor formalism and allows one to "see" things that are otherwise hidden among complicated expressions involving the six functors. I will demonstrate this by showing that a surprising amount of constructions and dualities in the literature admit a unified description in terms of the 2-category of kernels, including constructible sheaves, coherent sheaves, admissible smooth representations and Bernstein–Zelevinsky duality.

## Spherochromatism in Geometric Representation Theory

#### SANATH DEVALAPURKAR

#### Harvard University

#### Monday, June 9, 2025 @ 1:00 PM

Recent years have seen a flurry of developments relating homotopy theoretic constructions with results in arithmetic geometry and quantum field theory. In this talk, I will discuss another place where homotopy theory plays an important organizational role, namely geometric representation theory. Building on results relating commutative ring spectra k to the moduli stack of formal groups, I will explain some conjectures surrounding the "derived geometric Satake equivalence", which relates a category of sheaves of k-modules on the affine Grassmannian of a complex Lie group G to the algebraic geometry of its Langlands dual group  $\check{G}$ . The mechanism through which the Langlands dual side depends on k is determined entirely by the formal group corresponding to k. Time permitting, I will explain a related conjecture about the effect of loop-rotation equivariance, where the Langlands dual side is related to q-de Rham cohomology as recently introduced by Bhatt and Scholze in the case when the coefficients are connective complex K-theory.

## Involutions and the Brauer Group in Derived Algebraic Geometry

### LUCY YANG

#### Columbia University

Monday, June 9, 2025 @ 2:15 PM

Classical results of Albert and Saltman (extended by Knus, Parimala, Srinivas) have established a connection between the existence of (anti-)involutions on the Azumaya algebras A used to define the Brauer group and 2-torsion Brauer classes. Moreover, the presence of more general forms of involutions is related to the behavior of Brauer classes under corestriction along quadratic extensions. In this work, we investigate how the data of an involution on A is reflected in additional structure on its (derived) category of modules and introduce the notion of an involution on a generalized Azumaya algebra. Using the theory of Poincaré  $\infty$ -categories developed by Calmès,Dotto,Harpaz,Hebestreit,Land,Moi,Nardin,Nikolaus,Steimle, we introduce involutive versions of the Picard and Brauer group and relate them to their classical and non-involutive counterparts. This is joint work in progress with Viktor Burghardt and Noah Riggenbach.

## Microlocal Categorical Sheaves on Shifted Symplectic Spaces

#### ADEEL KHAN

#### Academia Sinica

Monday, June 9, 2025 @ 3:45 PM

I will describe a ladder of conjectural (n+1)-categorical invariants associated to n-shifted symplectic derived stacks. For n=0 these generalize categories of microsheaves on smooth symplectic schemes (closely related to Fukaya categories). For n=-1 and -2 they should recover categorifications of Donaldson-Thomas invariants of Calabi-Yau three- and four-folds, respectively. For n>0 they should be closely related to the higher Fukaya categories recently proposed by Pascaleff and Sibilla.

## Around Local Monodromy

### MARTIN GALLAUER

University of Warwick

Tuesday, June 10, 2025 @ 9:30 AM

Generically smooth families of algebraic varieties give rise to local monodromy operators in cohomology. In this talk I'll explore what lies behind this phenomenon: could it be that rigid-analytic geometry and homotopy theory are involved? At the end, I'll discuss some applications of this perspective to p-adic cohomology. (Based on joint work with Joseph Ayoub, Federico Binda, Alberto Vezzani.)

## Lax Higher Category Theory

### DAVID GEPNER

#### Johns Hopkins University

#### Tuesday, June 10, 2025 @ 11:00 AM

The Gray tensor product is an essential operation in higher category theory which systematically replaces the cartesian product. It forms a nonsymmetric biclosed monoidal structure on (infinity,infinity)-categories which corepresents functors and (op)lax natural transformations. It turns out that essentially all operations of interest in higher categorical settings are only functorial in this (op)lax sense, and that the (more) correct objects to study are categories enriched in (infinity,infinity)-categories under the Gray tensor product. We will explain this philosophy and some elementary notions in this setting, including fibrations and (op)lax (co)limits. This is joint work with Hadrian Heine.

## The Proetale Homotopy Type

#### PETER HAINE

University of California, Berkeley

Tuesday, June 10, 2025 @ 2:00 PM

Let X be a locally topologically noetherian scheme. In their paper on the proetale topology, Bhatt and Scholze defined the proetale fundamental group  $\pi_1^{\text{proét}}(X)$ . The profinite completion of  $\pi_1^{\text{proét}}(X)$  recovers the usual Etale fundamental group. Moreover,  $\pi_1^{\text{proét}}(X)$  agrees with  $\pi_1^{\text{ét}}(X)$ when X is normal, but  $\pi_1^{\text{proét}}(X)$  has the better property that it classifies  $\mathbb{Q}_{\ell}$ -local systems. In this talk, we will explain how to use condensed mathematics to define a proetale homotopy type whose fundamental group recovers the proetale fundamental group. We will also explain a number of computations of and foundational results about the proetale homotopy type. This is based on work with Barwick, Holzschuh, Lara, Mair, Martini, and Wolf.

## An Exceptional Inverse Image Functor for Sheaves of Categories

#### CARLOS DI FIORE

Universidad de Buenos Aires

Tuesday, June 10, 2025 @ 3:15 PM

I will introduce an exceptional inverse image functor for quasicoherent sheaves of categories together with its structural properties. We can play with this functor and smooth schemes to bring microlocalization and singular support ideas into the picture. There are two important examples, the formal neighborhood of a point in a smooth scheme and projective space. The first one will lead us into a version of higher Koszul duality and the second one into homological projective duality. This is joint work with G. Stefanich.

## Condensed Anima

#### LARS HESSELHOLT

University of Copenhagen/Nagoya University

Tuesday, June 10, 2025 @ 4:45 PM

My purpose in this talk is to make some propaganda for the Heyer–Mann six-functor formalism on condensed anima. The Clausen–Scholze infinity-category of condensed anima contains topological manifolds as static condensed anima and their underlying homotopy types as discrete condensed anima. Moreover, for every topological manifold Y, there is a map of condensed anima  $f: Y \to X$ to its underlying anima X, which is initial among maps of condensed anima from Y to a discrete condensed anima. (This map is secretly used to define the universal cover of a topological manifold.) I will explain the behavior of this map and the unique maps  $p: X \to 1$  and  $q \simeq pf: Y \to 1$  as far as Poincare duality in the Heyer–Mann six-functor formalism on condensed anima is concerned. As an application, I will show that, for Y compact, this naturally encodes the Bartels–Efimov–Nikolaus categorification of the assembly map in Waldhausen K-theory of X as well as the canonical lifting of the underlying anima X of Y to a simple anima. The condensed anima  $X \times Y$ , which is neither discrete nor static, will play a central role.

## Poincare Duality in Abstract 6-Functor Formalisms

#### BOGDAN ZAVYALOV

#### IAS/Princeton University

Wednesday, June 11, 2025 @ 9:00 AM

In this talk, I will discuss Poincare Duality in the context of abstract 6-functor formalisms. Somewhat surprisingly, a 6-functor formalism satisfies an appropriate form of Poincare Duality under a minimal set of extra assumptions. Furthermore, these assumptions are essentially independent of the "coefficient" categories D(X). This makes it easy to verify these assumptions in practice. In particular, this allows us to reprove previously established Poincare Duality results in a uniform and almost formal way.

## How to Enhance Categories, and Why

### DMITRY KALEDIN

Higher School of Economics

Wednesday, June 11, 2025 @ 10:00 AM

I am going to describe a framework for working with homotopically enhanced categories loosely based on Grothendieck's idea of a "derivator." The framework is manifestly model-independent, does not use the machinery of model categories, nor simplicial homotopy theory, and is pretty close to the usual categorical intuition and way of thinking.

## Grothendieck Classes of Twisted Symplectic Grassmannians

### GONCALO TABUADA

#### University of Warwick

#### Wednesday, June 11, 2025 @ 11:30 AM

The Grothendieck ring of varieties, introduced in a letter from Alexander Grothendieck to Jean-Pierre Serre (16 August 1964), plays an important role in algebraic geometry. However, despite the efforts of several mathematicians, the structure of this ring still remains poorly understood. In this talk, in order to better understand the Grothendieck ring of varieties, I will describe some new structural properties of the Grothendieck classes of twisted symplectic Grassmannians.

## Motivic Power Operations at the Characteristic

#### TONI ANNALA

#### University of Chicago

Thursday, June 12, 2025 @ 9:30 AM

Motivic power operations acting on the mod- $\ell$  motivic cohomology of smooth  $\mathbb{F}_p$ -schemes,  $\ell \neq p$ , were constructed by Voevodsky and played a key role in his proof of the Milnor and Bloch-Kato conjectures. In this talk, I describe joint work with Elden Elmanto, in which we extend Voevodsky's operations on mod-p motivic cohomology from characteristic 0 to characteristic p, thereby obtaining the long-sought-after motivic power operations at the characteristic. These operations satisfy all expected properties (except possibly generating, together with the Bockstein, all the endomorphisms of  $H\mathbb{F}_p$ ). If time permits, I will also discuss how to extend these operations to the motivic cohomology of singular  $\mathbb{F}_p$ -schemes, as recently defined by Elmanto-Morrow and Kelly-Saito. Additionally, I may mention other applications, such as defining obstructions to lifting motivic cohomology classes to algebraic cobordism classes, solving the motivic Steenrod problem for singular varieties at the characteristic, and identifying algebraic cycles that are not smoothable at the characteristic.

## Geometry of Singularities and Higher Structures

#### Fernando Muro

#### University of Seville

Thursday, June 12, 2025 @ 11:00 AM

I will outline a recent proof of the Donovan–Wemyss Conjecture, which strikingly links the birational geometry of threefold singularities with derived equivalences of finite-dimensional algebras. Central to this approach is a homological reinterpretation: contraction algebras arising from crepant resolutions of cDV singularities can be realized as derived endomorphism algebras of 2Z-cluster tilting objects in the singularity category. Leveraging the derived Auslander–Iyama correspondence, we deduce that derived equivalences of such algebras reflect isomorphisms of the underlying singularities. A key player in this story is the restricted universal Massey product, which acts as a fine invariant distinguishing derived structures. This is joint work with Gustavo Jasso and Bernhard Keller. This talk aims to showcase how geometry, representation theory, and higher algebra intertwine in proving a deep and elegant conjecture.

## Categorification of Six Functor Formalisms

#### GERMAN STEFANICH

Max-Planck Institute for Mathematics

Thursday, June 12, 2025 @ 2:00 PM

The goal of this talk is to explain how to associate, to any presentable six functor formalism Sh, a sequence of categorified formalisms nSh which we regard as theories of sheaves of (infty,n)categories of type Sh. In the same way that all the functoriality in Sh is concisely captured into a functor out of an infinity category of correspondences, its categorifications nSh give rise to functors out of (infty,n)-categories of correspondences. As an application I will explain a general recipe for constructing topological field theories of arbitrary dimension whose values on a manifold M are given by higher categories of sheaves on moduli stacks of locally constant maps from M into a fixed target.

## The Synthetic Hilbert Additive Group Scheme

#### ALICE HEDENLUND

Norwegian University of Science and Technology

Thursday, June 12, 2025 @ 3:15 PM

The Hilbert additive group arises in algebraic geometry as the affine group scheme coming from the free binomial ring on one generator. This can be endowed with the structure of a filtered stack by noting that the free binomial ring on one generator can be identified with integer-valued polynomials, which of course carries a canonical filtration by degree. This filtered stack is intimately related to the HKR filtration on Hochschild homology, as shown in work of Moulinos-Robalo-Toen. With a view towards geometrizing the motivic filtrations on topological Hochschild homology, we extend the filtered Hilbert additive group to the setting of spectral algebraic geometry to an object that we call the synthetic Hilbert additive group. This crucially relies on the even filtration introduced by Hahn-Raksit-Wilson and the view of even synthetic spectra as modules over the even sphere. This is joint work with Tasos Moulinos.

## TBD

#### Rok Gregoric

Johns Hopkins University

Thursday, June 12, 2025 @ 4:45  $\rm PM$ 

## TBD

### ARPON RAKSIT

Massachusetts Institute of Technology

Friday, June 13, 2025 @ 9:30 AM

## Sheared Prismatization

### AKHIL MATHEW

University of Chicago

Friday, June 13, 2025 @ 11:00 AM

I will discuss the theory of D-modules (i.e., modules over a ring of differential operators) in positive and mixed characteristic, and explain how the theory of prismatic cohomology (developed by Bhatt-Scholze, and extended by Bhatt-Lurie and Drinfeld) leads to new "deformations" of the category of D-modules (which in turn are related to p-adic Galois representations). Much of this talk will be expository, but the new results are joint work with Bhargav Bhatt, Artem Kanaev, Vadim Vologodsky, and Mingjia Zhang.

## The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

## List of Participants

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