Recent Trends in Homogeneous and Teichmuller Dynamics

September 23–27, 2024

About the Workshop

This workshop will bring together researchers from homogeneous dynamics, Teichmüller dynamics and closely related fields to discuss recent developments in these areas. The workshop aims at exploring analogies and connections between these fields with the goal of making progress on long standing open problems and questions. Of particular interest are the subjects of mixing, equidistribution, rigidity, flexibility, and applications, both from the qualitative and quantitative points of view.

Speakers

Nalini Anantharaman, Collège de France

Jon Chaika, University of Utah

Benjamin Dozier, Cornell University

Samantha Fairchild, MPI for Mathematics in the Sciences

Simion Filip, University of Chicago

Osama Khalil, University of Illinois Chicago

Georgios Kotsovolis, Princeton University

Elon Lindenstrauss, Hebrew University of Jerusalem

Amir Mohammadi, University of California San Diego

Hee Oh, Yale University

Wenyu Pan, University of Toronto

John Rached, Binghamton University

Omri Solan, Hebrew University of Jerusalem

Zhenqi Wang, Michigan State University

Andreas Wieser, Hebrew University of Jerusalem

Katy Woo, Princeton University

Lei Yang, Institute for Advanced Study

Organizers

Giovanni Forni
University of Maryland
Francisco Arana-Herrera
University of Maryland

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CSIC Building, 4th Floor
8169 Paint Branch Drive
University of Maryland
College Park, MD 20742

Contents

Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00					
	Breakfast				
9:00		Breakfast	Breakfast	Breakfast	Breakfast
	Mohammadi	Anantharaman	Khalil	Dozier	Yang
10:00					
	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
	Fairchild	Abou-Rached	Wang	Woo	Wieser
11:00					
	Discussion	Discussion		Discussion	Chaika
12:00	Lunch	Lunch		Lunch	
13:00					
14:00					
	Solan (3206 Kirwan Hall)	Pan (3206 Kirwan Hall)		Kotsovolis	
15:00					
	Coffee Break	Coffee Break		Coffee Break	
	Filip (3206 Kirwan Hall)	Lindenstrauss (3206 Kirwan Hall)		Oh	
16:00					
17:00					
18:00					

All talks will be held in the Brin MRC center, located on the fourth floor of the CSIC building.
Except for the Monday afternoon and Tuesday afternoon lectures which will take place in the Math Colloquium room, 3206 Kirwan Hall.

Workshop Overview

The main goal of this workshop is to bring together researchers from homogeneous dynamics, Teichmüller dynamics and closely related fields to discuss recent developments in these areas. The workshop aims at exploring analogies and connections between these fields with the goal of making progress on long standing open problems and questions. Of particular interest are the subjects of mixing, equidistribution, rigidity, flexibility, and applications, both from the qualitative and quantitative points of view.

Organizing committee

FRANCISCO ARANA-HERRERA, University of Maryland

GIOVANNI FORNI, University of Maryland

Workshop Schedule

Monday, September 23, 2024

8:20 - 8:50	Breakfast
8:50 - 9:00	Doron Levy (University of Maryland/Director, Brin MRC) *Opening*
9:00 - 10:00	Amir Mohammadi (University of California San Diego) *Dynamics on Homogeneous Spaces: a Quantitative Viewpoint*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Samantha Fairchild (Max Planck Institute for Mathematics in the Sciences) *Disjointness of Flows and Saddle Connections on Translation Surfaces*
11:30 - 12:00	Discussion
12:00 - 2:00	Lunch
2:00 - 3:00	Omri Solan (Hebrew University of Jerusalem) - 3206 Kirwan Hall *Gap in Critical Exponents of $SL_2(R)$ Orbits*
3:00 - 3:30	Coffee Break
3:30 - 4:30	Simion Filip (University of Chicago) - 3206 Kirwan Hall *Lyapunov Exponents, Higher Weight Hodge Structures, and Anosov Representations*

Tuesday, September 24, 2024

8:30 - 9:00   Breakfast

9:00 - 10:00   Nalini Anantharaman (College de France)
                 Spectral Gap of the Laplacian for Random Hyperbolic Surfaces

10:00 - 10:30   Coffee Break

10:30 - 11:30   John Abou-Rached (Binghamton University)
                 Critical exponents of Veech groups and closing lemmas in moduli space

11:30 - 12:00   Discussion

12:00 - 2:00   Lunch

2:00 - 3:00   Wenyu Pan (University of Toronto) - 3206 Kirwan Hall
                 Selberg, Ihara and Berkovich

3:00 - 3:30   Coffee Break

3:30 - 4:30   Elon Lindenstrauss (Hebrew University of Jerusalem) - 3206 Kirwan Hall
                 Time Change Rigidity of Unipotent Flows

Wednesday, September 25, 2024

8:30 - 9:00   Breakfast

9:00 - 10:00   Osama Khalil (University of Illinois Chicago)
Flattening, Mixing, and Fourier Decay

10:00 - 10:30   Coffee Break

10:30 - 11:30   Zhenqi Wang (Michigan State University)
Local Rigidity of Weak Or No Hyperbolicity Algebraic Actions

11:30 - 11:45   Group Photo

7:00 - 9:30   Conference Dinner

Thursday, September 26, 2024

8:30 - 9:00   Breakfast

9:00 - 10:00   Benjamin Dozier (Cornell University)
Counting Geodesics on Random Hyperbolic Surfaces

10:00 - 10:30   Coffee Break

10:30 - 11:30   Katy Woo (Princeton University)
Prime Number Theorems For Polynomials From Homogeneous Dynamics

11:30 - 12:00   Discussion

12:00 - 2:00   Lunch

2:00 - 3:00   Georgios Kotsovolis (Princeton University)
Bass Note Spectra of Binary Forms

3:00 - 3:30   Coffee Break

3:30 - 4:30   Hee Oh (Yale University)
L^2 Triviality of Discrete Subgroups

FRIDAY, SEPTEMBER 27, 2024

8:30 - 9:00   BREAKFAST

9:00 - 10:00   LEI YANG (Institute for Advanced Study)
                Effective Versions of Ratner's Equidistribution Theorem

10:00 - 10:30   COFFEE BREAK

10:30 - 11:30   ANDREAS WIESER (Hebrew University of Jerusalem)
                Effective Equidistribution of Orbits of Semisimple Groups

11:30 - 12:30   JON CHAIKA (University of Utah)
                Horocycle Flow on Strata of Translation Surfaces

Abstracts of talks

Dynamics on Homogeneous Spaces: a Quantitative Viewpoint

Amir Mohammadi

University of California San Diego

Monday, September 23, 2024 @ 9:00 AM

Rigidity phenomena in homogeneous spaces have been extensively studied over the past few decades with several striking results and applications. We will give an overview of activities pertaining to the quantitative aspect of the analysis in this context with an emphasis on recent developments.

Disjointness of Flows and Saddle Connections on Translation Surfaces

Samantha Fairchild

Max Planck Institute for Mathematics in the Sciences

Monday, September 23, 2024 @ 10:30 AM

A translation surface is a collection of polygons in the plane with parallel sides identified by translation to form a Riemann surface with a singular Euclidean structure. We will give a class of surfaces where we know that the geodesic flows are disjoint dynamical systems for almost every direction. The solution is connected to understanding pairs of saddle connections, which are a special type of closed geodesic. This is based on joint work with Claire Burrin and Jon Chaika.

Gap in Critical Exponents of $SL_2(R)$ Orbits

OMRI SOLAN

Hebrew University of Jerusalem

Monday, September 23, 2024 @ 2:00 PM

We will discuss the following result. For every nonarithmetic lattice $\Gamma < SL_2(C)$ there is ϵ_Γ such that for every $g \in SL_2(C)$ the intersection $g\Gamma g^{-1} \cap SL_2(R)$ is either a lattice or a has critical exponent $\delta(g\Gamma g^{-1} \cap SL_2(R)) \leq 1 - \epsilon_\Gamma$. This result extends Mohammadi-Margulis and Bader-Fisher-Milier-Strover. We will focus on an ergodic component of the proof, asserting certain preservation of entropy-contribution under limits of measures. If time permits I will discuss additional applications of the method.

Lyapunov Exponents, Higher Weight Hodge Structures, and Anosov Representations

SIMION FILIP

University of Chicago

Monday, September 23, 2024 @ 3:30 PM

Holomorphic differentials on Riemann surfaces lead to a weight 1 Hodge structure on the cohomology of the surface, which has been useful in studying the action of SL(2,R) on moduli spaces of holomorphic differentials. One such application is to establish a formula for the sum of the (positive) Lyapunov exponents for the diagonal group. I will discuss an extension of this formula to higher weight variations of Hodge structures, coming from families of Calabi–Yau manifolds. The formula is closely related to a dynamical property of the monodromy representation, namely that it is an Anosov representation. I will provide the necessary background for the relevant notions.

Spectral Gap of the Laplacian for Random Hyperbolic Surfaces

NALINI ANANTHARAMAN

College de France

Tuesday, September 24, 2024 @ 9:00 AM

Although there are several ways to "choose a compact hyperbolic surface at random", putting the Weil-Petersson probability measure on the moduli space of hyperbolic surfaces of a given topology is certainly the most natural. The work of M. Mirzakhani has made possible the study of this probabilistic model, providing exact formulas for certain integrals, as well as their asymptotic behaviour in the limit of large genus. I will be interested in the spectral gap λ_1 of the laplacian for a random compact hyperbolic surface, in the limit of large genus g : in joint work with Laura Monk, we show that asymptotically almost surely, $\lambda_1 > 1/4 - \epsilon$ for any $\epsilon > 0$. The proof relies on the trace method. We use asymptotic expansions in powers of g^{-1} for volume functions giving the distribution of the length spectrum, and prove that the coefficients possess the "Friedman-Ramanujan property" (a notion introduced by J. Friedman in his proof of the Alon conjecture for random regular graphs).

Critical exponents of Veech groups and closing lemmas in moduli space

JOHN ABOU-RACHED

Binghamton University

Tuesday, September 24, 2024 @ 10:30 AM

We report on recent work achieving analogs of closing lemmas that appear in homogeneous dynamics, in the setting of moduli space. We discuss and speculate on the role of understanding critical exponents of non-lattice Veech groups in obtaining these results more generally.

Selberg, Ihara and Berkovich

Wenyu Pan

University of Toronto

Tuesday, September 24, 2024 @ 2:00 PM

We use the Selberg zeta function to study the limit behavior of resonances of a degenerating family of Kleinian Schottky groups. We prove that, after a suitable rescaling, the Selberg zeta functions converge to the Ihara zeta function of a limiting finite graph associated with the relevant non-Archimedean Schottky group acting on the Berkovich projective line. Moreover, our techniques can be used to obtain effective statements. One key idea is to introduce an intermediate zeta function that captures Archimedean and non-Archimedean information (while the Selberg resp. Ihara zeta function concerns only Archimedean resp. non-Archimedean properties). This is a joint work with Jialun Li, Carlos Matheus, and Zhongkai Tao.

Time Change Rigidity of Unipotent Flows

Elon Lindenstrauss

Hebrew University of Jerusalem

Tuesday, September 24, 2024 @ 3:30 PM

Two non-isomorphic ergodic measure preserving flows can become isomorphic if one of the systems undergoes an appropriate time change. In this case we will say that these flows are Kakutani equivalent to each other. We say that an ergodic flow is loosely Kronecker if it is Kakutani equivalent to the straight line flow on (say) a two torus in an irrational direction (the exact direction is immaterial as these are all equivalent to each other). Landmark work of Ratner from the late 70s (that paved the way to her even more famous results on orbit closures and equidistribution of unipotent flows) establishes that 1) the horocycle flow on any finite area surface of constant negative curvature is loosely Kronecker. 2) the product of two such flows is not loosely Kronecker. It remained an open problem whether e.g. products of two horocycle flows are Kakutani equivalent to each other. In joint work with Daren Wei we show unipotent flows are very rigid under time changes, and indeed unless the flows are loosely Kronecker, two unipotent flows are Kakutani equivalent if and only if they are isomorphic as measure preserving flows.

Flattening, Mixing, and Fourier Decay

Osama Khalil

University of Illinois Chicago

Wednesday, September 25, 2024 @ 9:00 AM

We will discuss a result in additive combinatorics asserting that probability measures on Euclidean space enjoy polynomial Fourier decay outside a very sparse set of frequencies, unless large subsets of their supports concentrate near proper linear subspaces at many scales. The talk will focus on dynamical consequences of this statement towards mixing of geodesic flows, and rates of Fourier decay of large classes of dynamically defined measures. Parts of this work are joint with Simon Baker and Tuomas Sahlsten.

Local Rigidity of Weak Or No Hyperbolicity Algebraic Actions

Zhenqi Wang

Michigan State University

Wednesday, September 25, 2024 @ 10:30 AM

We study rigidity properties of abelian actions with weak or no hyperbolicty. We introduce a general strategy for proving C^∞ local rigidity of algebraic actions, which applies to non-tame systems. As a consequence, we show C^∞ local rigidity for a broad class of parabolic algebraic actions on homogeneous spaces of semisimple Lie groups. This is the first time in literature that (strong) local rigidity for these actions is addressed.

Counting Geodesics on Random Hyperbolic Surfaces

Benjamin Dozier

Cornell University

Thursday, September 26, 2024 @ 9:00 AM

There are several natural models of random hyperbolic surfaces: random covers of a fixed surface, random gluings of ideal triangles, and Weil-Petersson measure. The study of these is often motivated by a fruitful analogy with random regular graphs, which becomes stronger as the genus/number of vertices goes to infinity. I will discuss the problem of counting closed geodesics on these objects, and in particular the "birthday paradox", which concerns the length scale at which closed geodesics become very likely to self-intersect. A key tool is effective mixing of the geodesic flow and its relation to spectral gap of the Laplace operator. Based on joint work with Jenya Sapir.

Prime Number Theorems For Polynomials From Homogeneous Dynamics

Katy Woo

Princeton University

Thursday, September 26, 2024 @ 10:30 AM

The Bateman-Horn conjecture gives a prediction for how often an irreducible polynomial takes on prime values. In this talk, I will discuss the proof of Bateman-Horn for new classes of polynomials – for example, the determinant polynomial on nxn matrices and the determinant polynomial on nxn symmetric matrices. A key tool in the proof is the input of homogeneous dynamics to count the number of integral points on level sets. This talk is based on joint work with Giorgos Kotsovolis.

Bass Note Spectra of Binary Forms

Georgios Kotsovolis

Princeton University

Thursday, September 26, 2024 @ 2:00 PM

Mahler's program of determining the spectrum

$$\text{Spec}(P) := \left\{ \inf_{\underline{x} \in \Lambda \setminus \underline{0}} |P(\underline{x})|, \Lambda \subset \mathbb{R}^k \text{ a unimodular lattice} \right\}$$

for some homogeneous form P is central in the geometry of numbers and offers a unified framework for many unsolved problems in number theory and homogeneous dynamics. The case of binary quadratic forms is well studied and is classically related to the heights of geodesics on the modular surface. However, for binary forms of higher degrees much less is known. In 1940, Mordell determined the extremal lattices for the spectra of binary cubic forms and further conjectured the existence of spectral gaps. In this talk, we resolve this spectral problem for all binary cubic forms and more generally for all binary forms P of degree $n \geq 3$, by showing that quite contrary to these conjectures the spectrum $Spec(P)$ is always an interval.

L^2 Triviality of Discrete Subgroups

Hee Oh

Yale University

Thursday, September 26, 2024 @ 3:30 PM

For a discrete subgroup Gamma of a semisimple Lie group G, the space $L^2(Gamma \backslash G)$ is crucial in bridging representation theory with dynamics. We define Gamma to be L^2-trivial if $L^2(Gamma \backslash G)$ is weakly equivalent to $L^2(G)$. Determining L^2-triviality for a given discrete subgroup is challenging. We present a criterion for Anosov subgroups to be L^2 trivial based on the Hausdorff dimension of their limit sets. For example, we obtain that the image of any surface subgroup in $SL(n, R)$ under a positive representation is L^2-trivial for $n > 2$. This talk is based on joint works with Sam Edwards and with Subhadip Dey and Dongryul Kim.

Effective Versions of Ratner's Equidistribution Theorem

LEI YANG

Institute for Advanced Study

Friday, September 27, 2024 @ 9:00 AM

In this talk, I will talk about recent progress on proving effective versions of Ratner's equidistribution theorem for unipotent orbits in homogeneous spaces. I will mainly explain the proof for unipotent orbits in $SL(3, R)/SL(3, Z)$ and its application to the effective Oppenheim conjecture.

Effective Equidistribution of Orbits of Semisimple Groups

ANDREAS WIESER

Hebrew University of Jerusalem

Friday, September 27, 2024 @ 10:30 AM

Effective unipotent dynamics has been one of the most active areas within homogeneous dynamics in the past decade. In this talk, we will discuss effective equidistribution results for periodic orbits of semisimple groups. In a breakthrough from around 15 years ago, Einsiedler, Margulis, and Venkatesh established such a result whenever the group has finite centralizer in the ambient semisimple group. Using, in particular, an effective closing lemma from work with Lindenstrauss, Margulis, Mohammadi, and Shah, we remove this assumption on the centralizer.

Horocycle Flow on Strata of Translation Surfaces

Jon Chaika

University of Utah

Friday, September 27, 2024 @ 11:30 AM

For about 2 decades the horocycle flow on strata of translation surfaces was studied, very successfully, in analogy with unipotent flows on homogeneous spaces. By work of Ratner, Margulis, Dani and many others, unipotent flows on homogeneous spaces have striking rigidity properties. Recently, the horocycle flow on strata of translation surfaces was shown to not have many of these rigidity properties: 1) There exist horocycle orbit closures with fractional Hausdorff dimension. 2) There exist points which do not equidistribute under the horocycle flow with respect to any measure. 3) There exist points which equidistribute distribute under the horocycle flow to a measure, but they are not in the topological support of that measure. On the other hand, in work in progress we are showing that horocycle orbits on certain nice subloci of strata of translations surfaces are nice. This is joint work with J. Smillie and B. Weiss, O. Khalil and J. Smillie and B. Weiss and F. Ygouf.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

JOHN ABOU-RACHED, Binghamton University
NALINI ANANTHARAMAN, College de France
FRANCISCO ARANA-HERRERA, University of Maryland
JON CHAIKA, University of Utah
EMILIO CORSO, The Pennsylvania State University
BENJAMIN DOZIER, Cornell University
SAMANTHA FAIRCHILD, Max Planck Institute for Mathematics in the Sciences
SIMION FILIP, University of Chicago
GIOVANNI FORNI, University of Maryland
SAM FREEDMAN, University of Chicago
OSAMA KHALIL, University of Illinois Chicago
DONGRYUL KIM, Yale University
GEORGIOS KOTSOVOLIS, Princeton University
DORON LEVY, University of Maryland/Director, Brin MRC
ELON LINDENSTRAUSS, Hebrew University of Jerusalem
HOWARD MASUR, University of Chicago
AMIR MOHAMMADI, University of California San Diego
HEE OH, Yale University
CARLOS OSPINA, University of Utah
JOSHUA PAIK, The Pennsylvania State University
WENYU PAN, University of Toronto
ANTHONY SANCHEZ, University of California San Diego
PRATYUSH SARKAR, University of California San Diego
PETER SARNAK, Princeton University
JOHN SMILIE, University of Warwick
OMRI SOLAN, Hebrew University of Jerusalem
ZHENQI WANG, Michigan State University
ANDREAS WIESER, Hebrew University of Jerusalem
KATY WOO, Princeton University
LEI YANG, Institute for Advanced Study