BRIN MATHEMATICS RESEARCH CENTER

Macroscopic Behavior of Deterministic Systems

September 9-12, 2024

Participants

Peter Balint, BME
Giulia Basti, Gran Sasso Science Institute
Giovanni Canestrari, University of Roma Tor Vergata
Roberto Castorrini, University of Pisa
Serena Cenatiempo, Gran Sasso Science Institute
Stefan De Bievre, Université de Lille
Jacopo De Simoi, University of Toronto
Dmitry Dolgopyat, University of Maryland
Benjamin Doyon, King's College
Gregory Eyink, John Hopkins University
Pablo Ferrari, Universidad de Buenos Aires
Nicholas Fleming, University of Toronto
Pedro Garrido, Universidad de Granada
François Huveneers, King's College
Alessandra Iacobucci, University of Paris Dauphine
Tomasz Komorowski, Polish Academy of Sciences
Maxence Phalempin, University of Florence
Antonio Ponno, University of Padua
Hayate Suda, Keio University
Domokos Szasz, BME
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CSIC Building, 4th Floor
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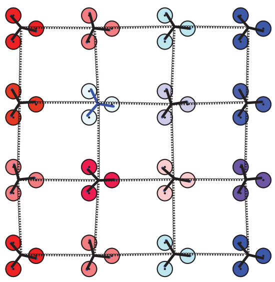
About the workshop

The dynamics of the world can be described in many different mathematical ways. In particular, macroscopically as the evolution of a continuous medium via partial differential equations or integral equations, or microscopically via the evolution of its constituents (molecules, atoms, etc.). The compatibility of such different descriptions is a wide-open mathematical problem. This workshop aims to bring together people working on such problems from many different angles to promote exchanges of ideas and the opening of new lines of research.



Organizers

Carlangelo Liverani, University of Tor Vergata and University of Maryland
Stefano Olla, University of Paris Dauphine and GSSI l'Aquila

DEPARTMENT OF MATHEMATICS

Contents

Schedule at a Glance

Time	Monday	Tuesday	Wednesday	Thursday
8:30		Breakfast	Breakfast	Breakfast
9:00	Toth	Cenatiempo	De Simoi	Doyon
10:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30	Canestrari	Basti	Szasz	Suda
11:30	Komorowski	De Bievre	Paradela Diaz	Ferrari
12:30	Lunch	Lunch	Lunch	
14:00	Garrido	Ponno	Balint	
15:00	Eyink	Coffee Break	Coffee Break	
15:30		Huveneers		
16:00	Coffee Break			
16:30	Informal Discussion on Macro vs. Micro			

Workshop Overview

The dynamics of the world can be described in many different mathematical ways. In particular, macroscopically as the evolution of a continuous medium via partial differential equations or integral equations, or microscopically via the evolution of its constituents (molecules, atoms, etc.). The compatibility of such different descriptions is a wide-open mathematical problem. This Workshop aims to bring together people working on such problems from many different angles to promote exchanges of ideas and the opening of new lines of research.

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Carlangelo Liverani, University of Tor Vergata & University of Maryland

Stefano Olla, University of Paris Dauphine & GSSI l'Aquila

Workshop Schedule

Monday, September 9, 2024

8:30 - 8:50 Breakfast

8:50 - 9:00 Sandra Cerrai (University of Maryland)
Opening

9:00 - 10:00 Balint Toth (Alfred Renyi Institute of Mathematics and University of Bristol)
The Random Lorentz Gas - Invariance Principle Beyond the Kinetic Time Scales

10:00 - 10:30 Coffee Break

10:30 - 11:30 Giovanni Canestrari (Univeristy of Roma Tor Vergata)
Heat Equation From a Deterministic Dynamics

11:30 - 12:30 Tomasz Komorowski (Polish Academy of Sciences)
On the Conversion of Work into Heat: Microscopic Models and Macroscopic Equations

12:30 - 2:00 Lunch

2:00 - 3:00 Pedro Garrido (Universidad de Granada)
Computer Simulations of Hard Particles: A Tool to Understand Macroscopic Laws

3:00 - 4:00 Gregory Eyink (John Hopkins University)
A Counterexample to the Hydrodynamic Scaling Limit and Landau-Lifschitz Fluctuating Hydrodynamics

4:00 - 4:30 Coffee Break

4:30 - 5:30 Informal Discussion on Macro vs. Micro

Tuesday, September 10, 2024

8:30 - 9:00	Breakfast
9:00 - 10:00	Serena Cenatiempo (Gran Sasso Science Institute) *Macroscopic Behaviour of Dilute Bose Gases: the Gross-Pitaevskii Equation*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Giulia Basti (Gran Sasso Science Institute) *Trial States for Dilute Gases of Hard-Core Bosons*
11:30 - 12:30	Stefan De Bievre (Universite de Lille) *Approach to Equilibrium: Simple Classical and Quantum Models*
12:30 - 2:00	Lunch
2:00 - 3:00	Antonio Ponno (University of Padua) *Quantum Dynamics of Trapped Boson Cold Atoms*
3:00 - 3:30	Coffee Break
3:30 - 4:30	Francois Huveneers (King's College) *Absence of Normal Transport in an Interacting Disordered Spin Chain*

Wednesday, September 11, 2024

8:30 - 9:00	Breakfast
9:00 - 10:00	Jacopo De Simoi (University of Toronto) *Statistical Properties of Some Mostly Expanding Fast-Slow Partially Hyperbolic Systems*
10:00 - 10:30	Coffee Break
10:30 - 11:30	Domokos Szasz (Budapest University of Technology and Economics) *The Rare Interaction Limit of a Hamiltonian System*
11:30 - 12:30	Jaime Paradela Diaz (University of Maryland) *KAM stability of large N-Body Problems*
12:30 - 2:00	Lunch
2:00 - 3:00	Peter Balint (Budapest University of Technology and Economics) *On the Dynamics of Averaged Quantities in a Crack Evolution Model of Polygonal Tesselations*
3:00 - 3:10	Group Photo
3:10 - 3:40	Coffee Break
7:00 - 9:00	Conference Dinner

Thursday, September 12, 2024

8:30 - 9:00 Breakfast

9:00 - 10:00 Benjamin Doyon (King's College)
Emergent Hydrodynamics in Integrable Many-Body Systems

10:00 - 10:30 Coffee Break

10:30 - 11:30 Hayate Suda (Keio University)
Scaling Limits for Solitons in the Randomized Box-Ball System

11:30 - 12:30 Pablo Ferrari (Universidad de Buenos Aires)
Hard Rod Hydrodynamics and the Levy Chentsov Field

12:30 - 12:45 Workshop Closing

Abstracts of talks

The Random Lorentz Gas - Invariance Principle Beyond the Kinetic Time Scales

Balint Toth

Alfred Renyi Institute of Mathematics and University of Bristol

Monday, September 9, 2024 @ 9:00 AM

Since the pioneering work of Paul and Tatiana Ehrenfest (1912) the deterministic (Hamiltonian) motion of a point-like particle exposed to the action of a collection of fixed, randomly located short range scatterers has been a much studied model of physical diffusion under fully deterministic (Hamiltonian) dynamics, with random initial conditions. This model of physical diffusion is known under the name of "random Lorentz gas" or "random wind-tree model". Celebrated milestones on the route to better mathematical understanding of this model of true physical diffusion are the Kinetic Limits for the tagged particle trajectory under the so-called Boltzmann-Grad (a.k.a. low density), or weak coupling approximations [Gallavotti (1970), Spohn (1978), Boldrighini-Bunimovich-Sinai (1982), respectively, Kesten-Papanicolaou (1980)]. Once the kinetic limits are established, under a second diffusive space-time scaling limit the central limit theorem (CLT) and invariance principle (IP) for the tagged particle motion follow. However, the CLT/IP under bare diffusive space-time scaling (without first applying the kinetic approximations) remains a Holy Grail.
In recent work we have obtained some intermediate results, partially interpolating between the two-steps-limit (first kinetic, then diffusive - as described above) and the bare-diffusive-limit (Holy Grail). We establish the Invariance Principle for the tagged particle trajectories under a joint kinetic+diffusive limiting procedure, performed simultaneously rather than successively, reaching significantly longer time scales than any earlier result (like, e.g., Komorowski-Ryzhik (2006) in classical weak coupling setting). The main ingredient is a coupling of the Hamiltonian trajectory (with random initial conditions) and an approximate Markovized version of the motion, and probabilistic and geometric controls on the efficiency of this coupling.
The Holy Grail remains, however, beyond reach.

Heat Equation From a Deterministic Dynamics

Giovanni Canestrari

Univeristy of Roma Tor Vergata

Monday, September 9, 2024 @ 10:30 AM

Deriving macroscopic hydrodynamic equations starting from the knowledge of the dynamics of the smaller constituents is a major problem in mathematics and physics. After a brief introduction, we show how it is possible to derive the heat equation for the thermal energy under diffusive space-time scaling for a purely deterministic microscopic dynamics satisfying Newton equations. More specifically, we study the evolution of the energy for a harmonic chain perturbed by an external chaotic force acting like a magnetic field.
This is joint work with Carlangelo Liverani and Stefano Olla.

On the Conversion of Work into Heat: Microscopic Models and Macroscopic Equations

Tomasz Komorowski

Polish Academy of Sciences

Monday, September 9, 2024 @ 11:30 AM

Nature has a hierarchical structure with macroscopic behavior arising from the dynamics of atoms and molecules. The connection between different levels of the hierarchy is however not always straightforward, as seen in the emergent phenomena, such as phase transition and heat convection. Establishing in a mathematical precise way the connection between the different levels is the central problem of rigorous statistical mechanics. One of the methods leading to such results is to introduce some stochasticity inside the system.
We summarise some of the results obtained recently concerning the derivation of the macroscopic heat equation from the microscopic behaviour of a harmonic chain with a stochastic perturbation. We focus our attention on the emergence of macroscopic boundary conditions.
The results have been obtained in collaboration with Joel Lebowitz, Stefano Olla, Marielle Simon.

Computer Simulations of Hard Particles: A Tool to Understand Macroscopic Laws

Pedro Garrido

Universidad de Granada

Monday, September 9, 2024 @ 2:00 PM

Hard particle systems are among the most successful, inspiring, and prolific models in physics. They contain the essential ingredients needed to understand a large class of complex phenomena. Nowadays, they can be used to provide insights into the macroscopic behavior of systems out of equilibrium. In this talk, we review some of our work in this direction, ranging from local equilibrium and Fourier's law to the transition to convective flow in the presence of gravity, the efficiency of boundary dissipation, and Boltzmann's entropy behavior.

A Counterexample to the Hydrodynamic Scaling Limit and Landau-Lifschitz Fluctuating Hydrodynamics

Gregory Eyink

John Hopkins University

Monday, September 9, 2024 @ 3:00 PM

We discusss a very simple counterexample to the idea that deterministic PDEs arise from microscopic models as a law of large numbers in a hydrodynamic scaling limit. The counterexample is a small vial of water, 5cm on a side, at room temperatue, very gently stirred. Elementary physical arguments suggest that thermal fluctuation effects should appear at about 1mm and this expectation is confirmed by numerical simulations. Simple application of hydrodynamic scaling for this system shows that deterministic Navier-Stokes only becomes valid in extreme, physically unrealistic situations. Instead, the correct physical description of the fluid appears to be given by nonlinear Landau-Lifschitz fluctuating hydrodynamics. This model has no clear meaning as a continuum stochastic PDE, but is it is mathematically well-defined as a low-wavenumber effective field theory, just as it is ordinarily derived from microscopic models by formal methods in the physics literature. Thus, we argue that the resolution of the famous Hilbert 6th Problem is that the "laws of motion of continua are not in fact valid for atomistic systems.

Macroscopic Behaviour of Dilute Bose Gases: the Gross-Pitaevskii Equation

Serena Cenatiempo

Gran Sasso Science Institute

Tuesday, September 10, 2024 @ 9:00 AM

Since the early experiments on Bose-Einstein condensation in cold atomic gases, the Gross-Pitaevskii equation has emerged as a unique tool for describing both the equilibrium and dynamical properties of dilute Bose gases at low temperature. From a mathematical perspective, pioneering works by Lieb, Seiringer and Yngvason (2000) and Erdos, Schlein and Yau (2010) have shown that the Gross-Pitaevskii equation can be rigorously derived from the many-body Schrodinger equation in a suitable scaling limit, known as the Gross-Pitaevskii regime. In this regime N interacting bosons are trapped in a region with volume of order one, and interact through a two body potential whose scattering length is of order 1/N, and N tends to infinity.

In this talk we discuss recent methods, developed since 2019, for characterizing fluctuations around the effective description provided by the Gross-Pitaevskii equation. These methods - valid for integrable interactions - offer a rigorous implementation of an heuristic theory due to Bogoliubov (1957), in a regime where Bogoliubov's approximations do not hold.

Based on joint works with C. Boccato, C. Brennecke, C. Caraci, B. Schlein.

Trial States for Dilute Gases of Hard-Core Bosons

Giulia Basti

Gran Sasso Science Institute

Tuesday, September 10, 2024 @ 10:30 AM

The (far from trivial) rigorous implementation of Bogoliubov theory developed by Boccato, Brennecke, Cenatiempo and Schlein (2019) provided an enhanced understanding of the low energy spectral properties of dilute bosonic gases in the Gross-Pitaevskii regime characterized by a scattering length of the order 1/N where the number of particles N tends to infinity. Despite this advance the highly correlated case of hard-core interactions was not covered in the analysis. In particular the Bogoliubov approach does not provide a good trial state since it does not preserve the hard-core domain. At the same time the Jastrow ansatz, successfully used by Dyson (1957) to get the leading order of the energy in the hard-core case, proves to be difficult to be handled at higher order of precision. In this talk we show how to combine the two approaches to get an upper bound on the ground state energy of hard-core bosons matching the expression for integrable interactions. An analogous upper bound in the thermodynamic limit (where the number of particles and the size of the box are sent to infinity keeping the density fixed) is still missing for hard-core . However, we show how the Jastrow factor has been recently used to get a new upper bound resolving the energy up to an error of the order of the expected next-to-leading-order correction predicted by Lee, Huang and Yang (1947) and recently obtained as a lower bound by Fournais and Solovej (2021).

Based on joint works with S. Cenatiempo, A. Giuliani, A. Olgiati, G. Pasqualetti, B. Schlein.

Approach to Equilibrium: Simple Classical and Quantum Models

Stefan De Bievre

Universite de Lille

Tuesday, September 10, 2024 @ 11:30 AM

Rigorous derivations of the approach of individual elements of large isolated systems to a state of thermal equilibrium, starting from arbitrary initial states, are exceedingly rare. We will present such models, one classical, one quantum. In both cases, the results allow us to establish that, through a mechanicsm of repeated scattering, one element of such a large system can, at small coupling, reach thermal equilibrium, even if the other elements of the system are not, themselves in equilibrium, as is the case in the study of return to equilibrium. We will further discuss a model for a freely expanding gas which also displays a phenomenon of approach to equilibrium.
The talk is based on joint work with Paul Parris, and with Marco Merkli.

Quantum Dynamics of Trapped Boson Cold Atoms

Antonio Ponno

University of Padua

Tuesday, September 10, 2024 @ 2:00 PM

The quantum dynamics of the Bose-Hubbard (BH) model with a finite number of sites is proven to be close to that of the discrete Bogoliubov-Gross-Pitaevskii (dBGP) equation over times that are exponentially long in the inverse of the temperature, as the latter goes to zero. The j-th component of the complex vector satisfying the dBGP equation turns out to be the quantum expectation of the j-th Dirac annihilator on the coherent initial state distributed according to a suitable invariant probability measure. A general estimate controlling the evolution of any polynomial observable is provided. The results hold independently of the physical dimension d of the lattice.

Absence of Normal Transport in an Interacting Disordered Spin Chain

Francois Huveneers

King's College

Tuesday, September 10, 2024 @ 3:30 PM

Many-body localization (MBL) is an out-of-equilibrium phase of matter featuring emergent integrability: There exists a complete set of local integrals of motion. As a result, an MBL system remembers its initial state for arbitrarily long times if the system is thermally isolated. This implies, in particular, a total absence of transport. Proving this at the mathematical level of rigor has proven very challenging and remains unresolved. In this talk, I will present a theorem stating that the diffusion constant of such systems vanishes, indicating that transport is at most sub-diffusive. An interesting aspect of the proof is that it relies on establishing MBL in some portions of the chains that are immune from resonances. Additionally, it rules out some numerical results that suggested MBL would not exist at all. Our work represents thus a step forward in rigorously establishing the existence of the MBL phase in one-dimensional systems.
From a work with W. De Roeck, L. Giacomin and O. Prosniak.

Statistical Properties of Some Mostly Expanding Fast-Slow Partially Hyperbolic Systems

Jacopo De Simoi

University of Toronto

Wednesday, September 11, 2024 @ 9:00 AM

In this joint work with K. Fernando and N. Fleming-Vazquez, we consider a class of sufficiently smooth two-dimensional fast-slow partially hyperbolic systems. This class is obtained by perturbing a trivial extension of a one-parameter family of one-dimensional expanding maps.
We assume that the averaged system has exactly one sink and that both Lyapunov exponents of the system are positive. Using an elaboration on the technique of standard pairs, we will show that, for sufficiently small perturbations, there exists a unique SRB measure for the system; we also show that the system exhibits exponential decay of correlations for Holder observables with explicit, nearly optimal bounds.
This work is a "mostly expanding" counterpart of the analogous result for "mostly contracting" systems (i.e. with one negative Lyapunov exponent) which was studied in a joint work with C. Liverani several years ago.

The Rare Interaction Limit of a Hamiltonian System

Domokos Szasz

Budapest University of Technology and Economics

Wednesday, September 11, 2024 @ 10:30 AM

In 2008 Gaspard and Gilbert came up with a Newtonian billiard particle model and, moreover, presented a strategy for the derivation of Fourier's 1807 law of heat conduction. The mesoscopic limit of their system is its rare interaction limit that is expected to be a Markov jump process for the energies of the particles. For reducing the dimension of their system, in 2016, we suggested the disk-piston model. Its rare interaction limit - also a Markov jump process - will be given by also showing how, in the proof, an additional reduction of the dimension is of help.
The results are joint with Peter Balint, Thomas Gilbert,
Peter Nandori and Imre Peter Toth.

KAM stability of large N-Body Problems

Jaime Paradela Diaz

University of Maryland

Wednesday, September 11, 2024 @ 11:30 AM

We consider the motion of an infinite number of particles, with masses $\{m_n\}_{n\in\mathbb{N}}$ satisfying $m_1 = 1$ and $m_n \leq \varepsilon e^{-n}$ for $n > 1$, which interact through Newtonian gravitation. We prove that, for $0 < \varepsilon \ll 1$ there exist initial conditions of this (infinite-dimensional) dynamical system leading to collisionless almost-periodic motions for which all the particles are confined in some bounded domain of the plane.
This is joint work with D. Dolgopyat and B. Fayad.

On the Dynamics of Averaged Quantities in a Crack Evolution Model of Polygonal Tesselations

Peter Balint

Budapest University of Technology and Economics

Wednesday, September 11, 2024 @ 2:00 PM

Polygonal tesselations, that is, tilings of the plane by convex polygons, are often used to describe geological pattern formation. In this talk I would like to present a model for the crack evolution of these structures, and to show that, in an appropriate limit of this model, the dynamics of averaged quantities follows a system of ODE.
This is joint work with Gabor Domokos and Krisztina Regos.

Emergent Hydrodynamics in Integrable Many-Body Systems

Benjamin Doyon

King's College

Thursday, September 12, 2024 @ 9:00 AM

The hydrodynamic theory emerging at large scales in integrable many-body systems forms a universality class of its own, referred to as generalised hydrodynamics (GHD). It is based on the idea of local relaxation to generalised Gibbs ensembles. It is expected to apply to a panoply of systems, including for instance the classical hard rods, quantum Lieb-Liniger model, quantum XXZ spin chain, and KdV soliton gas. I will review the main general equations at the Euler scale, their meaning, and their specialisation to various models. Then, first, I will discuss a new family of classical integrable particle models, which generalise the hard rods by admitting a two-body scattering shift function that can be chosen at will. I will explain rigorous results about their thermodynamics and hydrodynamics. They form a canonical family of models for the kinetic interpretation of GHD, and are connected to the gas of wave packets in the Lieb-Liniger and other Bethe-ansatz integrable models. Second, using such a kinetic picture, I will explain how diffusive corrections work, in particular how the standard Navier-Stokes-like diffusive correction is in fact only approximately valid at small macroscopic times, and what replaces it at all times. The new diffusive-scale equation is conjecturally valid for every integrable many-body system.
This is based on works with Leonardo Biagetti, Jacopo De Nardis, Friedrich Hubner and Takato Yoshimura.

Scaling Limits for Solitons in the Randomized Box-Ball System

Hayate Suda

Keio University

Thursday, September 12, 2024 @ 10:30 AM

The box-ball system (BBS) is a cellular automaton that exhibits the solitonic behavior. In recent years, with the rapid progress in the study of the hydrodynamics of integrable systems, there has been a growing interest in BBS with random initial distribution. In this talk, we consider the scaling limits for the tagged soliton in the BBS starting from certain stationary distribution.
This is an ongoing work with Stefano Olla and Makiko Sasada.

Hard Rod Hydrodynamics and the Levy Chentsov Field

Pablo Ferrari

Universidad de Buenos Aires

Thursday, September 12, 2024 @ 11:30 AM

A Poisson line process is the image of a two-dimensional Poisson process under the map $(a, b) \mapsto (at + b)_{t \in \mathbb{R}}$. By associating a step with each line of the process, a random surface is obtained. The diffusive rescaling of the surface converges to the Levy-Chentsov field, a classical Gaussian field.
A hard rod is a segment contained in $\mathbb{R}$ that travels ballistically until it collides with another hard rod, at which point they interchange positions. Identifying lines with hard rod ballistic displacement, and associating surface steps with hard rod jumps, we obtain hydrodynamics of the hard rods in the Euler and diffusive scalings, as a consequence of the rescaling properties of Poisson processes.
This is a survey of works with Chiara Franceschini, Dante Grevino, Herbert Spohn and Stefano Olla.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

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